Intrabeam Scattering in the VLHC

Valeri Lebedev

Fermilab

VLHC mini-workshop
SLAC, March 21-23, 2001.

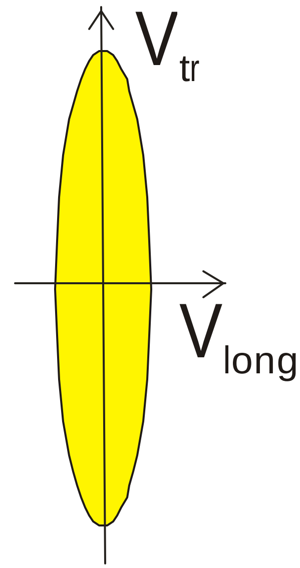
Content

VLHC Parameters

		Stage 1	Stage 2
Beam energy	E_p	20 TeV	87.5 TeV
Luminosity	L	10^{34} cm^{-2}s^{-1}	$2\cdot10^{34}$ cm^{-2}s^{-1}
Magnetic field	B_0	1.96 T	10 T
Injection energy	E_{inj}	0.9 TeV	10 TeV
Bunch spacing	t_b	18.9 ns	
Circumference	C=2pR	232 km	
Revolution frequency	f_0	1294 Hz	
Number of bunches	N_b	36943	
RF frequency	$\mathrm{f_{RF}}$	477.9 MHz	
Betatron tunes	$n_\perp$	~214	
Momentum compaction	a	$2.1\cdot10^{-5}$	
Beta-function at IP	b^*	30 cm	50 cm
Head-on beam-beam tune shift per IP	x	$1.8\cdot10^{-2}$	$1.8\cdot10^{-2}$
Number of particles per bunch	N	$2.5\cdot10^{10}$	
Beam current	I_b	0.190 A	0.175 A
RF voltage per turn, top/injection energy	V_0	50/50 MeV	50/50 MeV
Synchrotron frequency, top/injection energy	f_s	2.32/10.9 Hz	1.1/3.28 Hz
Rms momentum spread, top/injection energy	σ_p	$1.5/14.9\cdot10^{-4}$	$0.5/2.4\cdot10^{-4}$
Rms bunch length, top/injection energy	σ_s	6.6/14.2 cm	4.5/7.8 cm

1. Intrabeam scattering theory

If in the beam frame the longitudinal momentum spread is much less than the transverse one



IBS calculations can be significantly simplified

The IBS growth rate for longitudinal degree of freedom

- IBS transfers the energy from the transverse degrees of freedom to the longitudinal one and the growth rate can be approximated by the following formula

$$\frac{d}{dt}\left(q_\parallel{}^2\right) \equiv \frac{d}{dt}\left(\frac{\overline{p_\parallel{}^2}}{p}\right) = \frac{e^4 N_i}{4\sqrt{2}m_p{}^2 c^3 g_i{}^3 b_i{}^3 s_{si}} \left\langle \frac{\Xi_\parallel\left(q_x, q_y\right)}{\sqrt{q_x{}^2 + q_y{}^2}} \frac{L_C}{s_x s_y} \right\rangle_s \quad ,$$

where averaging is performed along the beam orbit,

$$\Xi_\parallel(x,y) \approx 1 + \frac{\sqrt{2}}{p}\ln\left(\frac{x^2+y^2}{2xy}\right) - 0.055\left(\frac{x^2-y^2}{x^2+y^2}\right)^2 ,$$

$s_x = \sqrt{e_x b_y + D^2 q_\parallel{}^2}$, $s_y = \sqrt{e_y b_y}$, $q_x = \sqrt{e_x / b_x}$ and $q_y = \sqrt{e_y / b_y}$

- are the beam sizes and angular spreads along the ring, and
- L_c - is the Coulomb logarithm.

- In the smooth focusing approximation,
- for equal horizontal and vertical emittances, $e_x = e_y$,
- equal betatron tunes, $n_x = n_y$, and
- small contribution of energy spread into the beam size, $D\left(\Delta p / p\right) << s_x$,

$\Rightarrow$ The momentum spread growth rate can be written in the following form

$$\boxed{\Lambda_\parallel \equiv \frac{1}{q_\parallel{}^2}\frac{d}{dt}\left(q_\parallel{}^2\right) = \frac{e^4 N_i L_C}{8 m_p{}^2 c^3 g_i{}^3 b_i{}^3 s_{si} e_x{}^{3/2} q_\parallel{}^2}\sqrt{\frac{n_x}{R}} \quad .}$$

<u>The IBS growth rate for transverse degree of freedom</u>

- The heating of the longitudinal degree of freedom causes cooling for both transverse degrees of freedom;
- Additional mechanism heats the horizontal degree of freedom
  - ➢ At regions with non-zero dispersion, changes in longitudinal momentum change the particles reference orbits, which additionally excites the horizontal betatron motion,

$$\frac{de_x}{dt} = \left\langle A_x \frac{dq_\parallel{}^2}{dt} \right\rangle_s \qquad ,$$

where $\quad A_x = \dfrac{D^2 + \left(D' b_x + a_x D\right)^2}{b_x}$

Finally, one can write for the emittance growth rates

$$\begin{bmatrix} \Lambda_x \\ \Lambda_y \end{bmatrix} \equiv \frac{1}{e_{x,y}} \frac{de_{x,y}}{dt} = \frac{e^4 N_i}{8\sqrt{2} m_p{}^2 c^3 g_i{}^3 b_i{}^3 s_{si} e_{x,y}}$$

$$\left\langle \frac{1}{\sqrt{q_x{}^2 + q_y{}^2}} \frac{L_C}{s_x s_y} \begin{bmatrix} 2A_x \Xi_\parallel\left(q_x, q_y\right) - \dfrac{b_x}{g_i{}^2} \Xi_\perp\left(q_x, q_y\right) \\ -\dfrac{b_y}{g_i{}^2} \Xi_\perp\left(q_y, q_x\right) \end{bmatrix} \right\rangle_s \quad .$$

where

$$\Xi_\perp(x,y) \approx 1 + \frac{2\sqrt{2}}{p} \ln\left(\frac{\sqrt{3x^2 + y^2}}{2y^2} x \right) + \frac{0.5429 \ln(y/x)}{\sqrt{1 + \ln^2(y/x)}} \qquad .$$

- In the smooth focusing approximation,
- for equal horizontal and vertical emittances, $e_x = e_y$,
- equal betatron tunes, $n_x = n_y$, and
- small contribution of energy spread into the beam size, $D(\Delta p/p) << s_x$,

$\Rightarrow$ The emittance growth rate can be written in the following form

$$\boxed{\Lambda_x = \frac{e^4 N_i L_C}{16 m_p^2 c^3 g_i^3 b_i^3 s_{si} e_x^{5/2}} \sqrt{\frac{R}{n_x}} \left(\frac{2}{n_x^2} - \frac{1}{g_i^2} \right)}$$

2. Intrabeam Scattering Estimate for the First Stage

Intra-beam scattering at injection

$\theta_{trinj} = 3.177\times 10^{-6}$

$\theta_{trinj}\cdot\beta_x = 0.055$ cm

$\sigma_{sinj} = 15.221$ cm

$\Delta PoverP_{inj} = 1.555\times 10^{-3}$

$$\frac{\theta_{trinj}}{\Delta PoverP_{inj}}\cdot\gamma_{inj} = 1.959$$

$$L_c(\gamma_{inj},\sigma_{sinj}) = 25.437$$

$$\frac{T_{longit}(\gamma_{inj},\sigma_{sinj},\Delta PoverP_{inj})}{3600\cdot 24} = 64.396$$ days

$$\frac{T_{tr}(\gamma_{inj},\sigma_{sinj})}{3600\cdot 24} = 6.231$$ days

Intra-beam scattering at full energy

$\theta_{tr} = 6.738\times 10^{-7}$

$\theta_{tr}\cdot\beta_x = 0.012$ cm

$\sigma_s = 6.747$ cm

$\Delta PoverP = 1.5\times 10^{-4}$

$$\frac{\theta_{tr}}{\Delta PoverP}\cdot\gamma = 95.758$$

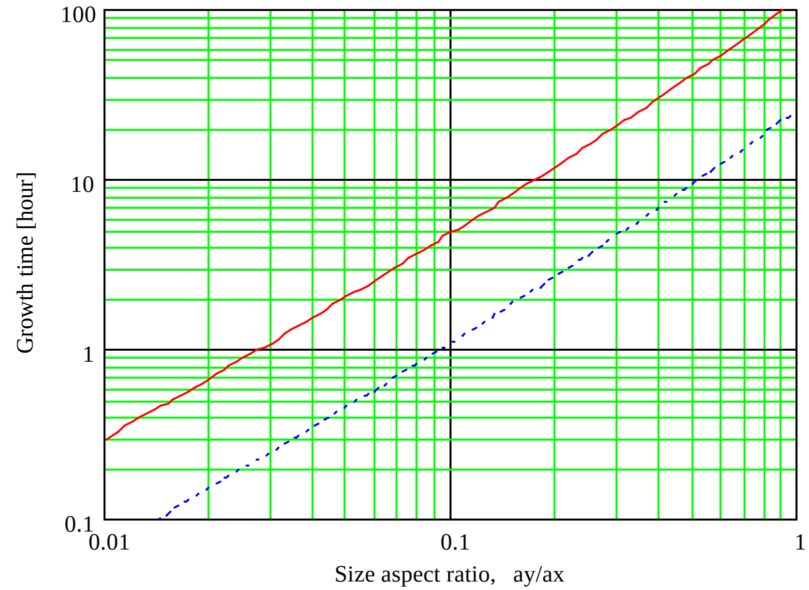
$$L_c(\gamma,\sigma_s) = 29.682$$

$$\frac{T_{longit}(\gamma,\sigma_s,\Delta PoverP)}{3600\cdot 24} = 23.846$$ days

$$\frac{T_{tr}(\gamma,\sigma_s)}{3600\cdot 24} = 11.019$$ days

3. Intrabeam Scattering Estimate for the Second Stage

- SR damping decreases beam sizes to the level where it will be stabilized by IBS or other heating mechanism
  - ➢ SR damping time is ~ 2 hour
- We need to prevent decrease of horizontal and longitudinal emittances to avoid stronger IBS and TMCI.
  - ▪ Vertical beam size decrease is limited by increase in IBS for horizontal degree of freedom. That happens at the size aspect ratio, s_y/s_x, of about 0.15.
    - Vertical emittance needs to be kept at this level to prevent growth of horizontal emittance.



Growth rates for the longitudinal momentum spread (red solid line) and horizontal beam size (blue dashed line) as function of beam size aspect ratio, s_y/s_x ; $s_{Dp/p} = 5\cdot10^{-5}$, $s_s = 2.6$ cm, $e_{nx} = 0.2$ mm mrad

-

4. Conclusions

- IBS does not contribute into emittance growth for the first stage
- IBS is one of limiting factors for the second stage after the SR damping will cause significant cooling of beam emittances